# Manuscript of Research Proposal: Graph of Supercalculi, conjugated calculi pairs, and Subcalculi

## **Introduction -**

This section may include:

§ What is to be done and the context of the project.

§ What is being done both generally and specifically in the same or related areas. (The reviewer should know that you know what is going on in the area in which you are proposing.)

§ An explanation and justification for unique or innovative approaches. (These are selling points about what makes your project special, unique and compelling and why it should be funded.)

## **Need Statement**

§ What needs to be done and why?

§ What significant needs are you trying to meet? Compared to other projects in the same area, what sets yours apart in terms of need?

§ What services are to be delivered? Why? Use specifics from preliminary studies, needs assessment, documentation, and data supporting your proposal.

§ What gaps that your work can fill exist in the knowledge base of your field?

**Gaps in the knowledge base**

The current understanding of sequent calculi and their relationships is fragmented and limited to specific systems or isolated aspects. There is a lack of a comprehensive framework that encompasses the diverse landscape of sequent calculi and their intricate connections. This research aims to address this gap by constructing a comprehensive graph of sequent calculi, providing a holistic view of the relationships and properties within this domain.

**Significant needs**

The development of novel logical systems and the advancement of automated reasoning techniques require a deeper understanding of the structural and semantic foundations of sequent calculi. A comprehensive graph of sequent calculi serves as a valuable tool for identifying patterns, exploring uncharted territories, and deriving new theorems within the realm of non-classical logic.

**Uniqueness of the approach**

This research distinguishes itself from existing work in its emphasis on the graph-based representation of sequent calculi. This approach offers a novel perspective on the relationships between different systems and facilitates the identification of hidden connections and patterns. Additionally, the focus on non-classical logics expands the scope of the research, addressing a less explored area of sequent calculi.

**Compared to other projects in the same area, what sets yours apart in terms of need?**

There have been several other projects that have proposed graph-based representations of sequent calculi. However, these projects have typically focused on specific aspects of sequent calculi, such as their structural properties or their computational complexity. This research proposal aims to develop a more comprehensive graph-based representation that can capture the full range of features and relationships between different sequent calculi.

In addition, this research proposal will develop a formal semantics for the graph-based representation, ensuring that it is consistent with the traditional semantics of sequent calculi. This will make the graph-based representation a more powerful tool for understanding and analyzing sequent calculi.

Finally, this research proposal will explore the expressiveness and limitations of the graph-based representation, identifying the types of logical systems that can be effectively represented and the limitations of this approach. This will provide valuable information for researchers who are considering using the graph-based representation for their own work.

**Overall, this research addresses a significant gap in the knowledge base of sequent calculi by providing a comprehensive and novel representation that can be leveraged to address important needs in the field of logic and automated reasoning.**

§ Is the problem both significant and manageable? Do you have the resources to handle the problem?

## **Goals and Objectives**

**Goal:** Establish a foundational understanding of the graph of sequent calculi for non-classical logics

**Objectives:**

1.1.1 Construct a comprehensive graph of sequent calculi for non-classical logics, capturing the structural and semantic relationships between different systems.

1.1.2 Analyze the graph to identify patterns, regularities, and connections that characterize the landscape of non-classical logics.

1.1.3 Develop a framework for navigating and interpreting the graph, enabling researchers to effectively explore and utilize this representation of non-classical logics.

**Activities:**

1.1.1.1 Gather and analyze existing sequent calculi for non-classical logics, encompassing a diverse range of systems and formalisms.

1.1.1.2 Identify the formal relationships between different sequent calculi, considering structural similarities, semantic equivalences, and embedding possibilities.

1.1.1.3 Represent the identified relationships as nodes and edges in a graph, creating a comprehensive visual representation of the landscape of non-classical logics.

1.1.2.1 Analyze the graph structure to identify patterns, regularities, and connections that reveal underlying principles and relationships within the realm of non-classical logics.

1.1.2.2 Investigate the implications of the identified patterns and relationships for the expressiveness, computational complexity, and decidability of non-classical logics.

1.1.2.3 Explore the potential of the graph to guide the development of new non-classical logics with tailored properties and applications.

1.1.3.1 Develop a formal framework for interpreting and navigating the graph, providing a structured approach to extracting information and insights from this representation.

1.1.3.2 Define graph-based metrics and measures to characterize the structural and semantic features of non-classical logics represented in the graph.

1.1.3.3 Create a user-friendly interface or tool that facilitates the exploration and analysis of the graph, enabling researchers to effectively utilize this resource.

**Measurement:**

The success of this research will be evaluated based on the following criteria:

* Comprehensiveness of the constructed graph of sequent calculi for non-classical logics
* Identification of novel patterns, regularities, and connections within the graph
* Development of a comprehensive framework for interpreting and navigating the graph
* Creation of graph-based metrics and measures for characterizing non-classical logics
* Development of a user-friendly interface or tool for exploring and analyzing the graph

**Outcomes:**

* A comprehensive graph of sequent calculi for non-classical logics, providing a visual representation of the relationships between different systems
* A deeper understanding of the structural and semantic landscape of non-classical logics, revealed through the analysis of the graph
* A framework for navigating and interpreting the graph, enabling researchers to effectively explore and utilize this representation
* Graph-based metrics and measures for characterizing non-classical logics
* A user-friendly interface or tool for exploring and analyzing the graph

**Products:**

* A research paper presenting the constructed graph, its analysis, and the developed framework for interpretation and navigation
* Open-source software or tools for visualizing and analyzing the graph
* Presentations at conferences and workshops to disseminate the findings and promote the utilization of the graph

**Goal:** Establish a comprehensive framework for identifying and classifying connections between sequent calculi

**Objectives:**

2.1.1 Develop a formal taxonomy of connections between sequent calculi, encompassing various types such as extensions, subtensions, conjugate duals, hyperextensions, hyposubtensions, supercalculi, subcalculi, hypocalculi, hypercalculi, and hyperduals.

2.1.2 Design a systematic methodology for identifying and classifying connections between sequent calculi based on their structural and semantic features.

2.1.3 Implement a computational tool or algorithm that automates the identification and classification of connections between sequent calculi.

**Activities:**

2.1.1.1 Analyze the existing literature on connections between sequent calculi to identify and categorize the different types of connections.

2.1.1.2 Define formal criteria and properties for each type of connection, ensuring a clear and consistent classification scheme.

2.1.1.3 Construct a hierarchical taxonomy of connections, representing the relationships and distinctions between different types.

2.1.2.1 Develop a systematic framework for analyzing the structural and semantic features of sequent calculi to identify potential connections.

2.1.2.2 Define formal procedures for identifying and classifying connections based on the identified structural and semantic features.

2.1.2.3 Validate the proposed methodology by applying it to a range of examples, ensuring its effectiveness and generality.

2.1.3.1 Design and implement a computational tool or algorithm that automates the identification and classification of connections based on the developed methodology.

2.1.3.2 Test and evaluate the computational tool or algorithm using a comprehensive set of examples, ensuring its accuracy and efficiency.

2.1.3.3 Develop a user-friendly interface or tool that facilitates the utilization of the computational tool or algorithm for researchers and practitioners.

**Measurement:**

The success of this research will be evaluated based on the following criteria:

* Comprehensiveness and clarity of the proposed taxonomy of connections between sequent calculi
* Effectiveness and generality of the systematic methodology for identifying and classifying connections
* Accuracy and efficiency of the computational tool or algorithm for automating the identification and classification of connections
* Usability and accessibility of the user-friendly interface or tool for researchers and practitioners

**Outcomes:**

* A formal taxonomy of connections between sequent calculi, providing a structured and comprehensive classification of different types of connections
* A systematic methodology for identifying and classifying connections between sequent calculi, enabling researchers to effectively analyze and categorize relationships between different systems
* A computational tool or algorithm that automates the identification and classification of connections between sequent calculi, streamlining the process and making it more accessible
* A user-friendly interface or tool that facilitates the utilization of the computational tool or algorithm, providing researchers and practitioners with a convenient and accessible resource

**Products:**

* A research paper presenting the proposed taxonomy, methodology, and computational tool or algorithm
* Open-source software or tools for identifying and classifying connections between sequent calculi
* Presentations at conferences and workshops to disseminate the findings and promote the utilization of the tools

**Goal:** Develop a systematic approach for designing novel logical calculi with tailored properties

**Objectives:**

3.1 Establish a framework for identifying and classifying the desired properties of novel logical calculi, considering factors such as expressiveness, computational complexity, decidability, and applications.

3.2 Develop a methodological framework for designing logical calculi with specific properties, utilizing graph-based representations and structural analysis techniques.

3.3 Implement a computational tool or algorithm that automates the design and verification of logical calculi with tailored properties.

**Activities:**

3.1.1 Analyze existing logical calculi and their properties to identify relationships between properties and structural features.

3.1.2 Define formal criteria and measures for evaluating the expressiveness, computational complexity, decidability, and applicability of logical calculi.

3.1.3 Develop a taxonomy of logical calculi based on their properties, enabling researchers to systematically explore and categorize different systems.

3.2.1 Design graph-based representations of logical calculi, capturing their structural relationships and highlighting potential property-determining features.

3.2.2 Develop structural analysis techniques for identifying and characterizing properties of logical calculi based on their graph representations.

3.2.3 Establish a systematic methodology for designing logical calculi with specific properties, utilizing graph-based representations and structural analysis techniques.

3.3.1 Design and implement a computational tool or algorithm that automates the design and verification of logical calculi with tailored properties.

3.1.3.2 Test and evaluate the computational tool or algorithm using a comprehensive set of examples, ensuring its accuracy and effectiveness in designing and verifying logical calculi with specific properties.

3.3.3 Develop a user-friendly interface or tool that facilitates the utilization of the computational tool or algorithm for researchers and practitioners.

**Measurement:**

The success of this research will be evaluated based on the following criteria:

* Comprehensiveness and clarity of the proposed framework for identifying and classifying properties of logical calculi
* Effectiveness and generality of the methodological framework for designing logical calculi with specific properties
* Accuracy and efficiency of the computational tool or algorithm for automating the design and verification of logical calculi with tailored properties
* Usability and accessibility of the user-friendly interface or tool for researchers and practitioners

**Outcomes:**

* A formal framework for identifying and classifying the properties of logical calculi, providing a structured and comprehensive approach to understanding and characterizing different types of properties
* A methodological framework for designing logical calculi with specific properties, enabling researchers to systematically design and develop logical systems with tailored characteristics
* A computational tool or algorithm that automates the design and verification of logical calculi with tailored properties, streamlining the process and making it more accessible
* A user-friendly interface or tool that facilitates the utilization of the computational tool or algorithm, providing researchers and practitioners with a convenient and accessible resource for designing logical systems with specific properties

**Products:**

* A research paper presenting the proposed framework, methodology, and computational tool or algorithm
* Open-source software or tools for designing and verifying logical calculi with tailored properties
* Presentations at conferences and workshops to disseminate the findings and promote the utilization of the tools

**Goal:** Explore and demonstrate the practical applications of the graph of sequent calculi

**Objectives:**

4.1.1 Identify potential applications of the graph of sequent calculi in various domains, including formal verification, automated reasoning, proof search, and logical programming.

4.1.2 Develop concrete use cases and examples that showcase the applicability of the graph in these domains, demonstrating its ability to solve practical problems and enhance existing techniques.

4.1.3 Evaluate the effectiveness and efficiency of the graph-based approach compared to traditional methods in addressing specific application scenarios.

**Activities:**

4.1.1.1 Conduct a comprehensive survey of existing applications of sequent calculi and related techniques to identify potential areas where the graph representation can be applied.

4.1.1.2 Collaborate with experts from different domains, such as software engineering, artificial intelligence, and computational linguistics, to explore potential applications in their respective fields.

4.1.1.3 Analyze the requirements and challenges of specific application domains to identify how the graph can be effectively utilized to address those challenges.

4.1.2.1 Develop concrete use cases for each identified application domain, providing detailed descriptions of how the graph can be used to solve specific problems or enhance existing techniques.

4.1.2.2 Implement prototypes or tools that demonstrate the practicality of the graph-based approach in the selected use cases.

4.1.2.3 Evaluate the performance and effectiveness of the graph-based approach compared to traditional methods in the chosen use cases.

4.1.3.1 Conduct rigorous testing and evaluation of the graph-based approach on a variety of benchmark problems and real-world applications.

4.1.3.2 Compare the computational complexity and resource requirements of the graph-based approach to traditional methods.

4.1.3.3 Analyze the qualitative aspects of the graph-based approach, such as its ability to provide insights, identify patterns, and simplify reasoning processes.

**Measurement:**

The success of this research will be evaluated based on the following criteria:

* Breadth and diversity of identified applications of the graph of sequent calculi
* Practicality and effectiveness of the graph-based approach in solving real-world problems
* Performance and efficiency of the graph-based approach compared to traditional methods
* Qualitative benefits of the graph-based approach, such as its ability to provide insights, identify patterns, and simplify reasoning processes

**Outcomes:**

* A comprehensive catalog of potential applications of the graph of sequent calculi in various domains
* Concrete use cases and examples that demonstrate the applicability of the graph in solving practical problems and enhancing existing techniques
* Benchmark results and performance evaluations comparing the graph-based approach to traditional methods
* Case studies and real-world applications that showcase the effectiveness and practicality of the graph-based approach

**Products:**

* A research paper presenting the identified applications, use cases, evaluations, and case studies
* Open-source software or tools that demonstrate the graph-based approach in specific application domains
* Presentations at conferences and workshops to disseminate the findings and promote the adoption of the graph-based approach

**Goal:** Establish a Formal Foundation for Graph-Based Representations of Sequent Calculi

**Objectives:**

5.1.1 Develop a rigorous mathematical framework for representing sequent calculi using graph theory concepts.

5.1.2 Define formal semantics for the graph-based representation of sequent calculi, ensuring consistency with the traditional semantics of sequent calculi.

5.1.3 Explore the expressiveness and limitations of the graph-based representation, identifying the types of logical systems that can be effectively represented and the limitations of this approach.

**Activities:**

5.1.1.1 Analyze the structural and semantic properties of sequent calculi to identify the key aspects that can be captured by graph-based representations.

5.1.1.2 Define graph-theoretic constructs and notations to represent the logical rules, formulas, and derivations of sequent calculi.

5.1.1.3 Formalize the relationships between graph-theoretic constructs and the corresponding components of sequent calculi.

5.1.2.1 Develop a formal interpretation of graph-based representations of sequent calculi, translating graph structures into logical formulas and derivations.

5.1.2.2 Verify the consistency of the graph-based semantics with the traditional semantics of sequent calculi, ensuring that the two approaches produce equivalent results.

5.1.2.3 Explore the implications of the graph-based semantics for the logical properties of sequent calculi, such as consistency, completeness, and decidability.

5.1.3.1 Investigate the range of logical systems that can be effectively represented using the graph-based approach, considering the expressiveness and limitations of graph theory.

5.1.3.2 Analyze the computational complexity of reasoning tasks, such as proof search and verification, within the graph-based representation.

5.1.3.3 Identify potential applications of the graph-based approach in various areas of formal logic and computer science, such as automated reasoning, metamathematics, and program analysis.

**Measurement:**

The success of this research will be evaluated based on the following criteria:

* Clarity and formality of the proposed mathematical framework for graph-based representations of sequent calculi
* Consistency and equivalence of the graph-based semantics with the traditional semantics of sequent calculi
* Expressiveness and limitations of the graph-based representation in capturing various types of logical systems
* Impact of the graph-based approach on computational complexity and reasoning tasks in formal logic
* Applicability of the graph-based approach in diverse areas of formal logic and computer science

**Outcomes:**

* A formal mathematical framework for representing sequent calculi using graph theory concepts
* A formal semantics for the graph-based representation of sequent calculi, ensuring consistency with traditional semantics
* An analysis of the expressiveness and limitations of the graph-based representation in capturing various types of logical systems
* Identification of potential applications of the graph-based approach in formal logic and computer science

**Products:**

* A research paper presenting the proposed framework, semantics, analysis, and potential applications
* Open-source software or tools for implementing and analyzing graph-based representations of sequent calculi
* Presentations at conferences and workshops to disseminate the findings and promote the adoption of the graph-based approach

Deliverables:

A comprehensive list of non-classical logics, including their axioms, rules, and properties.

A new formal framework for representing and reasoning about non-classical logics.

A new classification of non-classical logics.

A graph of sequent calculi that visually represents the relationships between these logics.

A list of common patterns and themes in the properties of non-classical logics.

A list of new areas of research suggested by the analysis of the graph of sequent calculi.

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### Goal 1: Investigating the Structural Properties of Sequent Calculi Using Graphs

**Subgoal 1.1: Develop a formal framework for representing sequent calculi using graphs**

* **Activities:**
  + **Conduct a thorough review of existing formal frameworks for representing logical systems, including graph-based approaches and algebraic representations.**
  + **Identify the key properties and requirements for a formal framework specifically tailored to sequent calculi, considering their unique structural and semantic features.**
  + **Design a novel graph-based formal framework that effectively captures the structure and semantics of sequent calculi, including their inferential rules, proof structures, and logical relationships.**
  + **Evaluate the expressiveness and flexibility of the proposed framework through the representation of a variety of sequent calculi, encompassing both well-established systems such as classical and intuitionistic sequent calculus, and newly developed sequent calculi with unique features.**
* **How the activities will be carried out:**
  + **The review of existing frameworks will involve examining academic literature, attending conferences, and consulting with experts in the field of sequent calculus and formal logic.**
  + **The identification of key properties and requirements will involve analyzing the characteristics of sequent calculi, considering their inferential mechanisms, proof structures, and logical properties, and the needs of researchers working with these systems.**
  + **The design of the novel framework will involve utilizing graph theory concepts, formal logic principles, and software engineering techniques, ensuring the framework can effectively represent the intricate relationships and interactions within sequent calculi.**
  + **The evaluation of the framework will involve representing a range of sequent calculi, including both well-established systems and newly developed ones, to assess its ability to capture their structural and semantic features.**

**Subgoal 1.2: Analyze the structural properties of sequent calculi using graph-based techniques**

* **Activities:**
  + **Apply the developed graph-based framework to analyze the structural properties of a range of sequent calculi, including their connectivity, symmetries, decomposition patterns, and relationships between different proof structures.**
  + **Identify and characterize common structural motifs and patterns that emerge across different sequent calculi, providing insights into their underlying computational properties and logical relationships.**
  + **Develop formal theorems that establish connections between the structural properties of sequent calculi and their logical properties, such as soundness, completeness, and decidability.**
* **How the activities will be carried out:**
  + **The analysis of structural properties will involve employing graph algorithms and techniques, such as graph isomorphism, subgraph detection, and graph decomposition, to examine the graph-based representations of sequent calculi.**
  + **The identification of common patterns will involve utilizing statistical analysis and pattern recognition methods to identify recurring structural motifs and patterns across different sequent calculi.**
  + **The development of formal theorems will involve rigorous mathematical proofs based on the formal framework, graph theory concepts, and logical reasoning principles, establishing connections between structural features and logical properties.**

**1.3: Develop graph-based algorithms for analyzing the structural properties of sequent calculi**

* **Activities:**
  + **Design and implement efficient graph algorithms specifically tailored to the analysis of sequent calculi, leveraging graph theory concepts and formal logic principles.**
  + **Optimize the developed algorithms for performance and scalability, considering the potentially large size and complexity of sequent calculus representations.**
  + **Integrate the algorithms into a software framework or toolset for analyzing sequent calculi, providing researchers with a user-friendly environment for exploring structural properties.**
* **How the activities will be carried out:**
  + **The design of algorithms will involve utilizing graph theory techniques such as graph traversal, pattern matching, and graph decomposition, tailoring them to the specific structure and semantics of sequent calculi.**
  + **The optimization of algorithms will involve employing techniques such as caching, data structures, and parallel processing, to improve their execution time and memory usage.**
  + **The integration of algorithms into a software framework will involve utilizing software engineering principles, modular design, and user interface considerations, to create a user-friendly and extensible toolset.**

**Subgoal 1.4: Apply graph-based algorithms to investigate the structural properties of sequent calculi**

* **Activities:**
  + **Utilize the developed graph-based algorithms to analyze the structural properties of a range of sequent calculi, including their connectivity, symmetries, decomposition patterns, and relationships between different proof structures.**
  + **Investigate the impact of structural properties on the computational behavior of sequent calculi, such as proof search efficiency and the complexity of proof structures.**
  + **Explore the application of graph-based techniques to the analysis of sequent calculus extensions and variations, such as modal sequent calculi and substructural sequent calculi.**
* **How the activities will be carried out:**
  + **The application of algorithms will involve executing the developed algorithms on the graph-based representations of sequent calculi, collecting and analyzing the resulting data.**
  + **The investigation of the impact of structural properties will involve correlating the identified structural features with the computational behavior of sequent calculi, using metrics such as proof length and search space size.**
  + **The exploration of extensions and variations will involve applying the graph-based techniques to the representations of these systems, analyzing their structural properties and identifying patterns.**

**Goal: Construct a comprehensive graph of sequent calculi and leverage it to identify uncharted non-classical languages and derive novel theorems within the realm of non-classical logic.**

**Objectives:**

**1.5.1 Construct a comprehensive graph of sequent calculi.**

**1.5.2 Identify uncharted non-classical languages based on the graph of sequent calculi.**

**1.5.3 Derive novel theorems within the realm of non-classical logic using the graph of sequent calculi.**

**Activities:**

**1.5.1.1 Gather and analyze existing sequent calculi.**

**1.5.1.2 Identify the relationships between different sequent calculi.**

**1.5.1.3 Represent the relationships between sequent calculi as a graph.**

**1.5.2.1 Analyze the graph of sequent calculi to identify potential new non-classical languages.**

**1.5.2.2 Formulate and investigate new non-classical languages based on the identified potential.**

**1.5.3.1 Extract novel theorems from the graph of sequent calculi.**

**1.5.3.2 Prove the validity of the extracted theorems.**

**Evaluation:**

**The success of this research will be evaluated based on the following criteria:**

* **The comprehensiveness of the graph of sequent calculi.**
* **The number of new non-classical languages identified.**
* **The novelty and significance of the derived theorems.**

**Timeline:**

**1.5.1.1 Gather and analyze existing sequent calculi: 6 months**

**1.5.1.2 Identify the relationships between different sequent calculi: 3 months**

**1.5.1.3 Represent the relationships between sequent calculi as a graph: 3 months**

**1.5.2.1 Analyze the graph of sequent calculi to identify potential new non-classical languages: 6 months**

**1.5.2.2 Formulate and investigate new non-classical languages based on the identified potential: 12 months**

**1.5.3.1 Extract novel theorems from the graph of sequent calculi: 6 months**

**1.5.3.2 Prove the validity of the extracted theorems: 12 months**

**Resources:**

**The research will require the following resources:**

* **Access to academic literature and databases**
* **Travel funds to attend conferences**
* **Collaborations with experts in the field of sequent calculi and non-classical logic**

**Expected Outcomes:**

* **A comprehensive graph of sequent calculi**
* **Identification of uncharted non-classical languages**
* **Derivation of novel theorems within the realm of non-classical logic**

**Dissemination:**

**The findings of this research will be disseminated through publications in peer-reviewed journals, presentations at conferences, and open-source software development.**

#### Goal: Leverage the graph of sequent calculi to identify uncharted non-classical languages and derive novel theorems within the domain of non-classical logic.

**Objectives:**

**1.6.1 Identify uncharted non-classical languages based on the graph of sequent calculi.**

**1.6.2 Derive novel theorems within the realm of non-classical logic using the graph of sequent calculi.**

**Activities:**

**1.6.1.1 Analyze the graph of sequent calculi to identify structural patterns and relationships that suggest the existence of uncharted non-classical languages.**

**1.6.1.2 Formulate hypotheses regarding the characteristics and properties of potential uncharted non-classical languages based on the identified patterns and relationships.**

**1.6.1.3 Investigate the formulated hypotheses by constructing formal representations and analyzing the logical properties of the proposed uncharted non-classical languages.**

**1.6.2.1 Identify promising areas within the graph of sequent calculi that exhibit potential for deriving novel theorems in non-classical logic.**

**1.6.2.2 Extract potential theorems from the identified promising areas by systematically examining the relationships and properties of sequent calculi within those regions.**

**1.6.2.3 Formalize and prove the extracted potential theorems using rigorous mathematical techniques and logical reasoning.**

**Evaluation:**

**The success of this research will be evaluated based on the following criteria:**

* **The number of uncharted non-classical languages identified.**
* **The novelty and significance of the identified uncharted non-classical languages.**
* **The number of novel theorems derived in non-classical logic.**
* **The novelty and significance of the derived novel theorems.**

**Timeline:**

**1.6.1.1 Analyze the graph of sequent calculi to identify structural patterns and relationships: 3 months**

**1.6.1.2 Formulate hypotheses regarding the characteristics and properties of potential uncharted non-classical languages: 3 months**

**1.6.1.3 Investigate the formulated hypotheses: 12 months**

**1.6.2.1 Identify promising areas within the graph of sequent calculi for deriving novel theorems: 3 months**

**1.6.2.2 Extract potential theorems from the identified promising areas: 3 months**

**1.6.2.3 Formalize and prove the extracted potential theorems: 12 months**

**Resources:**

**The research will require the following resources:**

* **Access to academic literature and databases**
* **Collaborations with experts in the field of sequent calculi and non-classical logic**
* **Access to computational resources for graph analysis and theorem proving**

**Expected Outcomes:**

* **Identification of uncharted non-classical languages with unique properties and applications**
* **Derivation of novel theorems that expand our understanding of non-classical logic and its capabilities**

**Dissemination:**

**The findings of this research will be disseminated through publications in peer-reviewed journals, presentations at conferences, and open-source software development.**

### Goal 2: Investigating the Relationship between Sequent Calculi and Substructural Logics

**Subgoal 2.1: Develop a formal framework for representing conjugated logic pairs using graphs**

* **Activities:**
  1. **Conduct a comprehensive review of existing formal frameworks for representing logical systems, including graph-based approaches and algebraic representations, with a particular focus on conjugated logic pairs.**
  2. **Identify the key properties and requirements for a formal framework specifically tailored to conjugated logic pairs, considering their unique structural and semantic features.**
  3. **Design a novel graph-based formal framework that effectively captures the intricate structure and semantics of conjugated logic pairs, including their ability to handle both classical and non-classical logics.**
  4. **Evaluate the expressiveness and flexibility of the proposed framework through the representation of a variety of conjugated logic pairs, encompassing both well-established systems and newly developed ones.**
* **How the activities will be carried out:**
  1. **The review of existing frameworks will involve examining academic literature, attending conferences, and consulting with experts in the field of conjugated logic pairs and formal logic.**
  2. **The identification of key properties and requirements will involve analyzing the characteristics of conjugated logic pairs, considering their ability to express both classical and non-classical logics, and the needs of researchers working with these systems.**
  3. **The design of the novel framework will involve utilizing graph theory concepts, formal logic principles, and software engineering techniques, ensuring the framework can effectively represent the complex relationships and interactions within conjugated logic pairs.**
  4. **The evaluation of the framework will involve representing a range of conjugated logic pairs, including both well-established systems such as classical logic and intuitionistic logic paired with their non-classical counterparts, and newly developed conjugated logic pairs with unique features.**

**Subgoal 2.2: Establish a connection between the graph-based representations of sequent calculi and conjugated logic pairs**

* **Activities:**
  1. **Analyze the graphical representations of sequent calculi and conjugated logic pairs to identify structural similarities and correspondences.**
  2. **Develop formal mappings between the graph-based representations of sequent calculi and conjugated logic pairs, preserving their structural and semantic properties.**
  3. **Utilize the established mappings to translate proofs and theorems between sequent calculi and conjugated logic pairs.**
  4. **Investigate the implications of these mappings on the relationship between the underlying logical systems.**
* **How the activities will be carried out:**
  1. **The analysis of graphical representations will involve employing graph comparison algorithms and techniques to identify common patterns and structural features.**
  2. **The development of formal mappings will involve constructing bijections or homomorphisms between the graph-based representations, ensuring that logical relationships are maintained.**
  3. **The translation of proofs and theorems will involve applying the established mappings to transform logical expressions and proof structures.**
  4. **The investigation of implications will involve analyzing the consequences of the mappings on the metatheoretical properties of the logical systems.**

### Secondary Objective: Harnessing the Power of Non-Classical Logics

### Goal 3: Investigating the Relationship between Sequent Calculi and Supercalculi

**Subgoal 3.1: Develop a formal framework for representing supercalculi using graphs**

* **Activities:**
  + Conduct a thorough review of existing formal frameworks for representing logical systems, including graph-based approaches and algebraic representations, with a particular focus on supercalculi.
  + Identify the key properties and requirements for a formal framework specifically tailored to supercalculi, considering their unique structural and semantic features.
  + Design a novel graph-based formal framework that effectively captures the intricate structure and semantics of supercalculi, including their ability to handle multiple modalities and non-standard connectives.
  + Evaluate the expressiveness and flexibility of the proposed framework through the representation of a variety of supercalculi, encompassing both well-established systems and newly developed ones.
* **How the activities will be carried out:**
  + The review of existing frameworks will involve examining academic literature, attending conferences, and consulting with experts in the field of supercalculi and formal logic.
  + The identification of key properties and requirements will involve analyzing the characteristics of supercalculi, considering their ability to express multimodal and non-classical logics, and the needs of researchers working with these systems.
  + The design of the novel framework will involve utilizing graph theory concepts, formal logic principles, and software engineering techniques, ensuring the framework can effectively represent the complex relationships and interactions within supercalculi.
  + The evaluation of the framework will involve representing a range of supercalculi, including both well-established systems such as modal logics and hybrid logics, and newly developed supercalculi with unique features.

### Goal 4: Investigating the Relationship between Sequent Calculi and Substructural Logics

**Subgoal 4.1: Develop a formal framework for representing substructural logics using graphs**

* **Activities:**
  + Conduct a comprehensive review of existing formal frameworks for representing substructural logics, including graph-based approaches and algebraic representations.
  + Identify the key properties and requirements for a formal framework specifically tailored to substructural logics.
  + Design a novel graph-based formal framework that effectively captures the structure and semantics of substructural logics.
  + Evaluate the expressiveness and flexibility of the proposed framework through the representation of a variety of substructural logics, including linear logic, relevance logic, and intuitionistic logic.
* **How the activities will be carried out:**
  + The review of existing frameworks will involve examining academic literature, attending conferences, and consulting with experts in the field.
  + The identification of key properties and requirements will involve analyzing the characteristics of substructural logics and the needs of researchers working with these systems.
  + The design of the novel framework will involve utilizing graph theory concepts, formal logic principles, and software engineering techniques.
  + The evaluation of the framework will involve representing a range of substructural logics, including both well-established systems and newly developed ones.

**Subgoal 4.2: Establish a connection between the graph-based representations of sequent calculi and substructural logics**

* **Activities:**
  + Analyze the graphical representations of sequent calculi and substructural logics to identify structural similarities and correspondences.
  + Develop formal mappings between the graph-based representations of sequent calculi and substructural logics, preserving their structural and semantic properties.
  + Utilize the established mappings to translate proofs and theorems between sequent calculi and substructural logics.
  + Investigate the implications of these mappings on the relationship between the underlying logical systems.
* **How the activities will be carried out:**
  + The analysis of graphical representations will involve employing graph comparison algorithms and techniques to identify common patterns and structural features.
  + The development of formal mappings will involve constructing bijections or homomorphisms between the graph-based representations, ensuring that logical relationships are maintained.
  + The translation of proofs and theorems will involve applying the established mappings to transform logical expressions and proof structures.
  + The investigation of implications will involve analyzing the consequences of the mappings on the metatheoretical properties of the logical systems.

### Foundational Objective: Laying a Rigorous Theoretical Framework

### Goal 5: Establishing a Formal Foundation for Sequent Calculi

**5.1: Develop a formal framework for representing sequent calculi using graphs**

* **Activities:**
  + Conduct a thorough review of existing formal frameworks for representing logical systems, including graph-based approaches and algebraic representations.
  + Identify the key properties and requirements for a formal framework specifically tailored to sequent calculi.
  + Design a novel graph-based formal framework that effectively captures the structure and semantics of sequent calculi.
  + Evaluate the expressiveness and flexibility of the proposed framework through the representation of a variety of non-classical logics.
* **How the activities will be carried out:**
  + The review of existing frameworks will involve examining academic literature, attending conferences, and consulting with experts in the field.
  + The identification of key properties and requirements will involve analyzing the characteristics of sequent calculi and the needs of researchers working with these systems.
  + The design of the novel framework will involve utilizing graph theory concepts, formal logic principles, and software engineering techniques.
  + The evaluation of the framework will involve representing a range of non-classical logics, including both well-established systems and newly developed ones.

**5.2: Use this framework to investigate the structural properties of sequent calculi**

* **Activities:**
  + Apply the developed formal framework to analyze the structural properties of a range of sequent calculi, including their connectivity, symmetries, and decomposition patterns.
  + Identify and characterize common structural motifs and patterns that emerge across different sequent calculi.
  + Develop formal theorems that establish relationships between the structural properties of sequent calculi and their logical properties, such as soundness and completeness.
* **How the activities will be carried out:**
  + The analysis of structural properties will involve applying graph algorithms and techniques to the graphical representation of sequent calculi.
  + The identification of common patterns will involve employing pattern recognition methods and statistical analysis.
  + The development of formal theorems will involve rigorous mathematical proofs based on the formal framework and logical principles.

**5.3: Apply this framework to the development of new non-classical languages**

* **Activities:**
  + Utilize the formal framework to design and construct novel non-classical logics with tailored properties and applications.
  + Employ the framework to systematically explore the space of possible non-classical logics, identifying new and interesting logical systems.
  + Formally analyze and evaluate the properties of the newly developed non-classical logics, including their expressiveness, consistency, and decidability.
* **How the activities will be carried out:**
  + The design of new logics will involve utilizing the framework's constructions to represent the desired logical properties and relationships.
  + The exploration of the space of non-classical logics will involve employing systematic search algorithms and techniques guided by the framework's constraints.
  + The formal analysis of newly developed logics will involve applying logical reasoning methods and tools to the framework's representation of these logics.

**Deliverables:**

* A comprehensive compendium of non-classical logics meticulously detailing their axioms, rules, and distinctive properties.
* A novel formal framework for representing and analyzing non-classical languages empowering a deeper understanding of their intricate constructs.
* A refined classification of non-classical logics organizing these diverse systems based on their inherent relationships and shared characteristics.
* A comprehensive graph of sequent calculi visually depicting the intricate web of connections between diverse non-classical logics.
* A catalog of common patterns and thematic elements that characterize the properties of non-classical logics providing insights into their unifying principles.
* An exhaustive list of new areas of research suggested by the analysis of the graph of sequent calculi opening up new frontiers for exploration in the realm of non-classical logic.

§ Goals statements identify the overall purpose of the project and a general indication of intent.

1. **Identification of non-classical logics:** A comprehensive list of non-classical logics will be compiled, including their axioms, rules, and properties.
2. **Classification of non-classical logics:** The non-classical logics will be classified based on their properties, such as the types of negation that they use, the consistency conditions that they satisfy, and the types of truth values that they employ.
3. **Construction of the graph of sequent calculi:** The graph of sequent calculi will be constructed by connecting the different non-classical logics based on their relationships.
4. **Analysis of the graph of sequent calculi:** The graph of sequent calculi will be analyzed to identify common patterns and themes, and to suggest new areas of research.
5. Develop the foundations and fundamentals for a universal theory of semantic languages.
6. Identify the precise metalinguistic conditions that define paraconsistent languages
7. Identify the precise metalinguistic conditions that define paracomplete languages.
8. Identify the precise metalinguistic conditions that define constructive languages.
9. Produce an automated reasoner that utilizes the union of paraconsistent constructions and paracomplete constructions in a metasystem of constructive proofs and constructive refutation refutations.
10. Schematically relate the subcalculi, supercalculi, hypocalculi, and hypercalculi graphs to universal quantum computing.

§ Objectives are action statements with measurable outcomes to be completed by a specified time and under specified conditions.

## **Approach/Methodology**

§ How are you going to carry out your project?

The graph will represent calculi as nodes and their relationships as edges, with edge types encoding different kinds of connections, such as equivalence, embedding, and extension.

**Literature Review:**

* Conduct an extensive review of relevant literature on sequent calculi including intuitionistic logic, counter-intuitionistic logic, linear logic, the logic of qubits, Ardeshir-Vaezian’s sequent calculus U, Sambin’s Basic Logic, and T-Norm hypersequent calculi.
* Criticize the fundamental concepts, principles, and applications of these calculi.
* Identify and analyze existing research related to the graph of supercalculi, conjugated calculi pairs, and subcalculi.

**2. Conceptual Framework Development:**

* Formulate a clear and precise conceptual framework for the graph of supercalculi, conjugated calculi pairs, and subcalculi.
* Define the key components and relationships within the graph structure.
* Establish a formal representation of the graph using appropriate mathematical and graphical notation.
* Establish a libre open source programming language capable of representing all supercalculi, conjugated calculi pairs, and subcalculi and their semantics.
* Establish an automated reasoning suite utilizing all sequent calculi in a modular way to reason about finite subgraphs of the graph of supercalculi, conjugated calculi pairs, and subcalculi.

**3. Metamathematical Analysis:**

* Employ metamathematical techniques to investigate the properties and structure of the graph of supercalculi, conjugated calculi pairs, and subcalculi.
* Analyze the graph's connectivity, paraconsistency, and paracompleteness.
* Explore the relationships between the graph's structure and the underlying logical systems.

**4. Metalinguistic Investigation:**

* Utilize metalinguistic tools to examine the expressive power and limitations of the graph of supercalculi, conjugated calculi pairs, and subcalculi.
* Analyze the graph's ability to represent and formalize various logical concepts and relationships.
* Represent the symmetries and dualities of not only the object languages but the metalanguages of the various calculi.
* Develop metalanguages for all semantic languages.
* Evaluate the graph's effectiveness in capturing the nuances of semantic languages.

**5. Comparative Analysis:**

* Compare and contrast the graph of supercalculi, conjugated calculi pairs, and subcalculi with alternative approaches to representing semantic languages.
* Identify the strengths and weaknesses of each approach in terms of expressiveness, conciseness, and computational efficiency.
* Discuss the implications of the graph-based approach for understanding and reasoning within and without semantic language.

§ What specific activities do you propose to meet the goals and objectives you have outlined, and how will those activities be carried out?

**Specific Activities:**

To achieve the outlined goals and objectives, the following specific activities will be undertaken:

* Gather and organize relevant literature on semantic language and graph of supercalculi, conjugated calculi pairs, and subcalculi..
* Construct a conceptual diagram or model to represent the graph of supercalculi, conjugated calculi pairs, and subcalculi.
* Develop formal definitions and theorems related to the graph's structure and properties.
* Utilize metamathematical tools, such as proof theory and model theory, to analyze the graph's behavior.
* Employ metalinguistic techniques to assess the expressive power and limitations of the graph.
* Conduct comparative analysis with alternative approaches to representing non-classical languages.
* Programmatically represent the works in a logical programming paradigm programming language in a published code repository.

## **Outcomes, Benefits, Results**

The project will deliver the following outcomes.

* A comprehensive list of non-classical logics, including their axioms, rules, and properties.
* A new formal framework for representing and reasoning about semantic languages.
* A new classification of semantic languages.
* A graph of sequent calculi that visually represents the relationships between these logics.
* A list of common patterns and themes in the properties of non-classical logics.
* A list of new areas of research suggested by the analysis of the graph of supercalculi, conjugated calculi pairs, and subcalculi.

The project will also have the following benefits.

* It will provide a better understanding of the logical relationships between different sequent calculi.
* It will facilitate the development of new and more powerful sequent calculi.
* It will enable the application of sequent calculi to new areas.

The project will also produce the following results.

* A new formal framework for representing and reasoning about semantic languages.
* A new classification of sublanguages, superlanguages, and hyperlanguages as well as subcalculi, supercalculi, and hypercalculi.
* A graph of supercalculi, conjugated calculi pairs, and subcalculi that visually represents the relationships between these logics.
* A list of common patterns and themes in the properties of non-classical logics and non-classical languages.
* A list of new areas of research suggested by the analysis of the graph of supercalculi, conjugated calculi pairs, and subcalculi.

§ Outcomes - What are the products of your work?

§ Impact - What are the benefits and results of your work?

§ Measurement - Can your outcomes, benefits and results be measured?

§ Products - What does the funding agency get in return for supporting your proposal?

## **Project Director/Principal Investigator and Staff**

§ List the qualifications and experience of the proposed project director/principal investigator.

§ List the qualifications and experience of key project staff.

**Primary Objective: Establishing a Unified Understanding of Non-Classical Logics**

**Gaps that this work can fill:**

* **There is a lack of efficient and scalable graph-based algorithms specifically designed for analyzing the structural properties of sequent calculi.**
* **The impact of structural properties on the computational behavior of sequent calculi remains incompletely understood.**
* **The applicability of graph-based techniques to the analysis of sequent calculus extensions and variations has not been fully explored.**

**Gaps that this work can fill:**

* **The current understanding of the structural properties of sequent calculi is limited and lacks a comprehensive and systematic approach to analysis.**
* **There is a need for rigorous and expressive graph-based techniques to effectively capture the intricate structure and relationships within sequent calculi.**
* **The identification of common structural patterns and their relationship to logical properties remains incompletely understood.**
* **The development of formal theorems that connect structural features to computational and logical properties can provide a deeper understanding of the nature of sequent calculi.**

**Gaps that this work can fill:**

* **The current understanding of the relationship between sequent calculi and conjugated logic pairs is limited and lacks a comprehensive formal framework for analysis.**
* **There is a need for a rigorous and expressive formal framework that can effectively capture the connections between the graphical representations of these systems.**
* **The precise relationship between the structural properties of sequent calculi and the logical features of conjugated logic pairs remains unclear.**
* **The implications of the relationship between sequent calculi and conjugated logic pairs on proof theory and automated reasoning have not been fully explored.**

**Gaps that this work can fill:**

* **The current understanding of formal frameworks for representing supercalculi is limited and lacks a unified and expressive approach.**
* **There is a need for a rigorous and flexible formal framework that can effectively capture the intricacies of supercalculi, including their multimodal and non-classical nature.**
* **The structural and semantic relationships between supercalculi and their underlying logical systems remain incompletely understood.**
* **The development of a graph-based formal framework can facilitate the analysis and comparison of different supercalculi and their applications.**

**Gaps that this work can fill:**

* The current understanding of the relationship between sequent calculi and substructural logics is limited and lacks a comprehensive formal framework for analysis.
* There is a need for a rigorous and expressive formal framework that can effectively capture the connections between the graphical representations of these systems.
* The precise relationship between the structural properties of sequent calculi and the logical features of substructural logics remains unclear.
* The implications of the relationship between sequent calculi and substructural logics on proof theory and automated reasoning have not been fully explored.

**Gaps that this work can fill:**

* The current understanding of formal frameworks for representing sequent calculi is fragmented and lacks a unified approach.
* There is a need for a rigorous and expressive formal framework that can effectively capture the intricacies of sequent calculi.
* The structural properties of sequent calculi remain largely unexplored, and their relationships to logical properties are not fully understood.
* The development of new non-classical logics is often ad hoc and lacks a systematic approach based on formal frameworks.

**Context of the project:**

* **Sequent calculi play a crucial role in formal logic, providing a powerful tool for representing and reasoning about logical systems.**
* **A comprehensive understanding of the structural properties of sequent calculi can provide insights into their computational efficiency, expressiveness, and limitations.**
* **The development of efficient and scalable graph-based algorithms can facilitate the systematic analysis of sequent calculi, leading to the discovery of new properties and optimization strategies.**
* **The exploration of the impact of structural properties and the application of graph-based techniques to extensions and variations can contribute to advancements in proof theory, automated reasoning, and the design of new logical systems.**

**Context of the project:**

* **Sequent calculi have emerged as powerful tools for formalizing and reasoning about a wide range of logical systems, with applications in proof theory, automated reasoning, and artificial intelligence.**
* **Establishing a comprehensive understanding of the structural properties of sequent calculi can provide insights into their computational behavior, expressiveness, and limitations.**
* **The development of graph-based techniques for analyzing sequent calculi can facilitate the identification of common patterns, the discovery of new properties, and the development of optimization strategies for automated reasoning systems.**
* **The establishment of formal connections between structural features and logical properties can contribute to advancements in proof theory, automated reasoning, and the design of new logical systems.**

**Context of the project:**

* **Sequent calculi and conjugated logic pairs are powerful tools for formalizing and reasoning about logical systems, with applications in diverse areas such as artificial intelligence, linguistics, and computer science.**
* **Establishing a clear connection between sequent calculi and conjugated logic pairs can provide a unified framework for understanding and analyzing these systems.**
* **The development of formal mappings between graphical representations can facilitate the translation of proofs and theorems, enhancing interoperability and cross-fertilization between the two domains.**
* **The investigation of the implications of these connections can lead to new insights into the nature of conjugated logic pairs and their applications in formal logic, artificial intelligence, and other areas.**

**Context of the project:**

* **Supercalculi have emerged as powerful tools for formalizing and reasoning about multimodal and non-classical logics, with applications in diverse areas such as artificial intelligence, linguistics, and computer science.**
* **Establishing a comprehensive formal framework for representing supercalculi using graphs can provide a unified foundation for understanding, analyzing, and comparing these systems.**
* **The development of formal mappings between graphical representations of supercalculi can facilitate the translation of proofs and theorems, enhancing interoperability and cross-fertilization between different supercalculi.**
* **The investigation of the implications of these connections can lead to new insights into the nature of supercalculi and their applications in formal logic, artificial intelligence, and other areas.**

**Context of the project:**

* Sequent calculi and substructural logics are powerful tools for formalizing and reasoning about logical systems with non-classical features.
* Establishing a clear connection between sequent calculi and substructural logics can provide a unified framework for understanding and analyzing these systems.
* The development of formal mappings between graphical representations can facilitate the translation of proofs and theorems, enhancing interoperability and cross-fertilization between the two domains.
* The investigation of the implications of these connections can lead to new insights into the nature of substructural logics and their applications in proof theory, automated reasoning, and other areas.

**Context of the project:**

* The research on sequent calculi has gained significant traction in recent years, driven by their applications in automated reasoning, proof theory, and artificial intelligence.
* The development of a formal framework for representing sequent calculi using graphs can provide a solid foundation for further advancements in this area.
* The investigation of the structural properties of sequent calculi can lead to a deeper understanding of their logical behavior and potential applications.
* The design of new non-classical logics using the formal framework can expand the repertoire of logical systems available for modeling and reasoning about complex phenomena.